Current algebra derivation of temperature dependence of hadron couplings with currents

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Abstract. The vector and the axial-vector meson couplings with the vector and the axial-vector currents respectively at finite temperature were obtained by calculating all the relevant one-loop Feynman graphs with vertices obtained from the effective chiral Lagrangian. On the other hand, the same couplings were also derived by applying the method of current algebras and the hypothesis of partial conservation of axial-vector currents. The latter method appears to miss certain terms; in the case of the vector meson coupling with the vector current, for example, a term containing the $\rho\omega\pi$ coupling is missed. A similar situation would also appear for the nucleon coupling with the nucleon current. In this note we resolve these differences.

1 Introduction

The thermal two-point functions of the vector and the axial-vector currents are evaluated in [1] from their one-loop Feynman graphs with vertices given by the effective chiral Lagrangian of strong interactions [2]. At low, but not too small, temperature T, the ρ meson coupling F_ρ with the vector current, for example, is found to depend on temperature as

$$F_\rho^T = F_\rho \left\{ 1 - \left(1 + \frac{g_1^2}{3} \right) \frac{T^2}{12 F_\pi^2} \right\}. \qquad (1.1)$$

Here F_π is the familiar pion decay constant, $F_\pi = 93\,\mathrm{MeV}$ and g_1 is the $\rho\omega\pi$ coupling, $g_1 = .87$. Although there is no strict power counting of momenta according to the number of loops when heavy mesons are present, the above result is expected to hold to within an accuracy of about 20% [2].

Previously, the authors of [3] evaluated the same quantities in a somewhat different way. At low temperature they expanded the thermal trace over states in the two-point functions, retaining only the vacuum and the one-pion state. The forward pion-current amplitude so obtained was apparently evaluated with the method of the current algebra and the method of the partially conserved axial-vector current (PCAC), reducing it to a combination of vacuum two-point functions of the vector and the axial-vector currents. It allowed them to conclude that the coupling F_ρ^T would change with temperature according to (1.1), but *without* the g_1^2 term.

A similar situation would arise also in the calculation of the temperature dependence of the nucleon coupling λ

with the nucleon current [4]. It has been calculated in [5] as

$$\lambda^T = \lambda \left\{ 1 - \frac{(g_A^2 + 1)}{32} \frac{T^2}{F_\pi^2} \right\}, \qquad (1.2)$$

by evaluating in the effective theory the one-loop Feynman graphs for the thermal two-point function of nucleon currents. Here g_A is the axial-vector coupling constant of the nucleon, $g_A = 1.26$. If, however, we follow the procedure of [3] to work out the same quantity by applying apparently the current algebra techniques, we would *miss* the g_A^2 term in (1.2).

The evaluation of the thermal part of one-loop Feynman graphs is actually a tree level calculation, as the corresponding part of the propagator of a particle (here pion) contains a mass-shell delta function. Now, at the tree level, the current algebra plus PCAC approach is strictly equivalent to the method of chiral perturbation theory [6]. One thus wonders why the two approaches would lead to different results for the temperature dependence of the couplings stated above.

Historically, the idea of PCAC was introduced [7] to extract the pion pole contribution to matrix elements of operators containing the axial-vector current. At small momentum k carried by the current, this is the only source of dominant contribution, as far as processes involving only pions (Goldstone bosons) are concerned. However, when there are heavy (non-Goldstone) particles in the process, there is another potential source of dominant contribution to such matrix elements, namely the graphs in which the axial-vector current attaches to an external heavy particle line. The additional heavy particle propagator so created can be singular at small momentum k. In fact, this is the mechanism contributing dominantly to processes of soft pion emission and absorption [8].

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In this note we rederive the temperature dependence of F_ρ^T and λ^T in the framework of current algebra in the chiral symmetry limit. We begin with the Ward identities for the relevant amplitudes involving the pion and the currents. We show that there are indeed heavy particle poles in the above two-point functions that are as dominant as the pion pole at small momentum k. In the case of the vector current, it is the ω meson pole in the neighborhood of the ρ meson pole, when we consider temperatures higher than the mass difference of the two mesons. In the case of a nucleon current, it is the nucleon pole itself, there being no mass difference here to deal with. It is these contributions, giving rise to the g_1^2 and the g_A^2 terms in (1.1) and (1.2), respectively, that are ignored in [3].

In Sects. 2 and 3 we present current algebra derivations of (1.1) and (1.2) respectively, where the contributions of the heavy particle poles are also calculated explicitly along with that of the pion pole. Our discussions are contained in Sect. 4.

2 Current algebra derivation of (1.1)

The derivation is based on the thermal two-point function of vector currents, $V_\mu^i(x), i = 1, 2, 3$, with which the ρ communicates. At low temperature $\beta^{-1} (= T)$, the heat bath is dominated by pions. Thus the leading correction to the ρ meson propagation is expected to come from its scattering off the pions. Accordingly the thermal trace is expanded as [3]

$$\mathrm{Tr} e^{-\beta H} T V_\mu^i(x) V_\nu^j(0) / \mathrm{Tr} e^{-\beta H}$$
$$= \langle 0| T V_\mu^i(x) V_\nu^j(0) |0\rangle \qquad (2.1)$$
$$+ \int \frac{\mathrm{d}^3 k\, n(k)}{(2\pi)^3\, 2|\boldsymbol{k}|} \sum_m \langle \pi^m(k)| T V_\mu^i(x) V_\nu^j(0) |\pi^m(k)\rangle_{\mathrm{conn}},$$

where $n(k) = (e^{\beta|\boldsymbol{k}|} - 1)^{-1}$ is the distribution function of the pions. T also denotes the time ordering of all operators that follow it. $\langle \cdots \rangle_{\mathrm{conn}}$ is the connected part of the matrix element. The first term gives the ρ pole in terms of its mass and coupling in vacuum. To find their changes at finite temperature to leading order, we use the method of current algebra to extract the leading terms of the pion matrix element at small k.

The appropriate Ward identity follows from the momentum space amplitude

$$W_{\alpha\mu\nu}^{mnij}(q, k, q', k')$$
$$= \int \mathrm{d}^4 z \mathrm{d}^4 x \mathrm{d}^4 x' e^{ik\cdot z} e^{iq\cdot x} e^{-iq'\cdot x'}$$
$$\times \langle 0| T A_\alpha^m(z) V_\mu^i(x) V_\nu^j(x') |\pi^n(k')\rangle, \qquad (2.2)$$

describing the scattering process shown in Fig. 1. As usual, we contract it with k^α and integrate by parts with respect to z and ignore the surface term to get

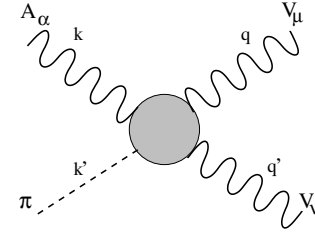$$k^\alpha W_{\alpha\mu\nu}^{mnij}(q, k, q', k')$$

Fig. 1. Amplitude $W_{\alpha\mu\nu}^{mnij}(q, k, q', k')$ describing the scattering $\pi(k') + V_\nu(q') \longrightarrow A_\alpha(k) + V_\mu(q)$. Isospin indices are omitted in the diagram

$$= i \int \mathrm{d}^4 z \mathrm{d}^4 x \mathrm{d}^4 x' e^{ik\cdot z} e^{iq\cdot x} e^{-iq'\cdot x'}$$
$$\times \partial_z^\alpha \langle 0| T A_\alpha^m(z) V_\mu^i(x) V_\nu^j(x') |\pi^n(k')\rangle. \qquad (2.3)$$

Since the axial-vector current is conserved in the chiral limit, the divergence operator gives non-zero contributions only when it acts on the theta functions in the definition of the time-ordered product, giving

$$\partial_z^\alpha \langle 0| T A_\alpha^m(z) V_\mu^i(x) V_\nu^j(x') |\pi^n(k')\rangle$$
$$= \delta(z_0 - x_0) \langle 0| T([A_0^m(z), V_\mu^i(x)] V_\nu^j(x')) |\pi^n(k')\rangle$$
$$+ \delta(z_0 - x_0') \langle 0| T([A_0^m(z), V_\nu^j(x')] V_\mu^i(x)) |\pi^n(k')\rangle.$$

The equal time commutators of currents are given by the current algebra,

$$[A_0^m(z), V_\mu^i(x)]_{z_0=x_0} = i\epsilon^{mil} A_\mu^l(x) \delta^3(z - x).$$

Also using translational invariance to remove one more integration in (2.3), we get the Ward identity as follows:

$$k^\alpha W_{\alpha\mu\nu}^{mnij}$$
$$= -(2\pi)^4 \delta^4(q + k - q' - k') \qquad (2.4)$$
$$\times \left[\epsilon^{mil} \int \mathrm{d}^4 x e^{i(q+k)\cdot x} \langle 0| T A_\mu^l(x) V_\nu^j(0) |\pi^n(k')\rangle \right.$$
$$\left. + \epsilon^{mjl} \int \mathrm{d}^4 x e^{iq\cdot x} \langle 0| T V_\mu^i(x) A_\nu^l(0) |\pi^n(k')\rangle \right].$$

To find the leading (constant) piece in $k^\alpha W_{\alpha\mu\nu}^{mnij}$ in the limit $k^\alpha \to 0$, we need only to calculate the terms in $W_{\alpha\mu\nu}^{mnij}$ that are singular in this limit. One such term is, of course, the π pole contribution, as shown in graph (a) of Fig. 2, that can be readily evaluated by inserting the pion intermediate state in the matrix element of (2.2),

$$k^\alpha W_{\alpha\mu\nu}^{mnij} \xrightarrow{\pi}$$
$$- (2\pi)^4 \delta^4(q + k - q' - k') F_\pi \qquad (2.5)$$
$$\times \int \mathrm{d}^4 x e^{iq\cdot x} \langle \pi^m(k)| T V_\mu^i(x) V_\nu^j(0) |\pi^n(k')\rangle,$$

where F_π is defined by

$$\langle 0| A_\alpha^m(z) |\pi^n(k)\rangle = i\delta^{mn} F_\pi k_\alpha e^{-ikz}.$$

As we shall show below, there is another finite contribution to $k^\alpha W_{\alpha\mu\nu}^{mnij}$ as $k^\alpha \to 0$ due to the ω pole in the vicinity
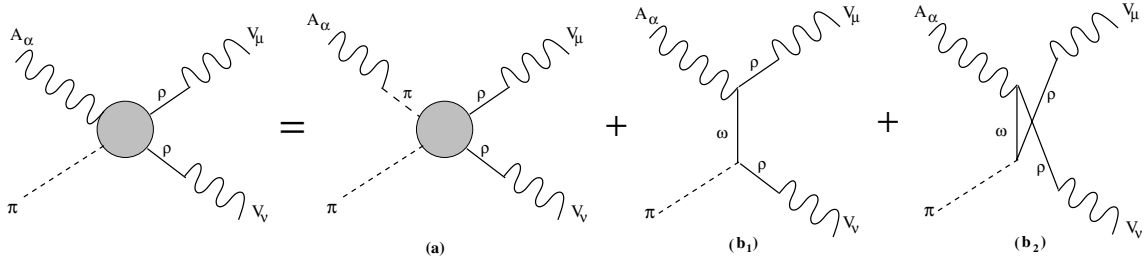
Fig. 2. Singular contributions to $W_{\alpha\mu\nu}^{mnij}(q, k, q', k')$ as $k, k' \to 0$, when the vector currents are coupled to ρ meson

of the ρ pole. Denoting this contribution as

$$k^\alpha W_{\alpha\mu\nu}^{mnij} \xrightarrow{\omega} (2\pi)^4 \delta^4(q + k - q' - k') k^\alpha R_{\alpha\mu\nu}^{mnij}(q, k, k'),$$

the Ward identity (2.4) gives the result

$$F_\pi \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}q \cdot x} \langle \pi^m(k)|TV_\mu^i(x)V_\nu^j(0)|\pi^n(k')\rangle$$

$$= \epsilon^{mil} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}(q+k)\cdot x} \langle 0|TA_\mu^l(x)V_\nu^j(0)|\pi^n(k')\rangle \qquad (2.6)$$

$$+ \epsilon^{mjl} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}q \cdot x} \langle 0|TV_\mu^i(x)A_\nu^l(0)|\pi^n(k')\rangle + k^\alpha R_{\alpha\mu\nu}^{mnij}.$$

The first two matrix elements on the right of (2.6) can be reduced to vacuum matrix elements, again by using Ward identities. Thus for the first one we define the amplitude

$$w_{\beta\mu\nu}^{mnij}(q, k, k') \qquad (2.7)$$

$$= \int \mathrm{d}^4 z' \mathrm{d}^4 x \mathrm{e}^{-\mathrm{i}k' \cdot z'} \mathrm{e}^{\mathrm{i}(k+q)\cdot x} \langle 0|TA_\mu^l(x)V_\nu^j(0)A_\beta^n(z')|0\rangle.$$

One proceeds exactly as before by contracting $w_{\beta\mu\nu}^{mnij}$ with k'^β. Here the pion pole is the only dominant contribution at small k', so that the Ward identity for this amplitude becomes

$$F_\pi \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}(k+q)\cdot x} \langle 0|TA_\mu^l(x)V_\nu^j(0)|\pi^n(k')\rangle$$

$$= \epsilon^{nll'} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}q' \cdot x} \langle 0|TV_\mu^{l'}(x)V_\nu^j(0)|0\rangle$$

$$+ \epsilon^{njl'} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}(q+k)\cdot x} \langle 0|TA_\mu^l(x)A_\nu^{l'}(0)|0\rangle. \quad (2.8)$$

A similar equation holds for the second matrix element on the right of (2.6). We thus get from (2.6) the pion matrix element in (2.1) as

$$\int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}qx} \sum_m \langle \pi^m(k)|TV_\mu^i(x)V_\nu^j(0)|\pi^m(k)\rangle$$

$$= -\frac{4}{F_\pi^2} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}q \cdot x} \langle 0|TV_\mu^i(x)V_\nu^j(0)|0\rangle$$

$$+ \frac{4}{F_\pi^2} \int \mathrm{d}^4 x \mathrm{e}^{\mathrm{i}q \cdot x} \langle 0|TA_\mu^i(x)A_\nu^j(0)|0\rangle$$

$$+ \frac{1}{F_\pi} k^\alpha R_{\alpha\mu\nu}^{ij}(q, k). \qquad (2.9)$$

The remaining task is to evaluate the ω meson pole contribution, shown in graphs (b$_1$) and (b$_2$) of Fig. 2. However, we do not evaluate them as Feynman graphs with the intermediate lines as propagators, but insert intermediate states in the spirit of the method of current algebra, as we already did with the pion pole extraction in (2.5). Noting the quantum numbers of ω, it is given by

$$W_{\alpha\mu\nu}^{mnij}(q, k, q', k') \xrightarrow{\omega}$$

$$\int \mathrm{d}^4 z \mathrm{d}^4 x \mathrm{d}^4 x' \mathrm{e}^{\mathrm{i}kz} \mathrm{e}^{\mathrm{i}qx} \mathrm{e}^{-\mathrm{i}q'x'} \sum_{\sigma_1} \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4}$$

$$\times \langle 0|TA_\alpha^m(z)V_\mu^i(x)|\omega(q_1, \sigma_1)\rangle$$

$$\times \langle \omega(q_1, \sigma_1)|V_\nu^j(x')|\pi^n(k')\rangle \frac{\mathrm{i}}{q_1^2 - m_\omega^2}$$

$$+\text{crossed term.} \qquad (2.10)$$

Its correction to the ρ pole is obtained by isolating this pole at the vector current vertices. Then extracting the coordinate dependence by translation invariance, we can integrate over the coordinates to get delta functions in momentum, which are then removed by intergals over the intermediate momenta. We thus get

$$W_{\alpha\mu\nu}^{mnij} \xrightarrow{\omega, \rho}$$

$$\sum_{\sigma_1, \sigma_2, \sigma_3, b, c} \langle 0|V_\mu^i(0)|\rho^b(q, \sigma_2)\rangle$$

$$\times \langle \rho^b(q, \sigma_2)|A_\alpha^m(0)|\omega(q + k, \sigma_1)\rangle$$

$$\times \langle \omega(q + k, \sigma_1)|\pi^n(k'), \rho^c(q', \sigma_3)\rangle$$

$$\times \langle \rho^c(q', \sigma_3)|V_\nu^j(0)|0\rangle$$

$$\times \frac{\mathrm{i}^3}{(q^2 - m_\rho^2)(q'^2 - m_\rho^2)\{(q + k)^2 - m_\omega^2\}}$$

$$+ \text{crossed term.} \qquad (2.11)$$

To evaluate the matrix elements in (2.11), we write the relevant pieces of the effective Lagrangian based on the chiral symmetry of QCD. In the notation of [2], these are

$$\mathcal{L}_{\mathrm{int}} = \frac{1}{2\sqrt{2}} \frac{F_V}{m_V} \langle \hat{V}_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{\sqrt{3}}{4} g_1 \epsilon_{\mu\nu\rho\sigma} \langle \hat{V}^{\mu\nu} \{\hat{V}^\rho, u^\sigma\} \rangle. \qquad (2.12)$$

Here the fields are all 3×3 octet matrices. $\hat{V}^\mu$ denotes the vector meson fields and $\hat{V}^{\mu\nu}$ their field strengths. The field strengths $f_+^{\mu\nu}$ (and $f_-^{\mu\nu}$) are those of the external fields,

v_μ^i and a_μ^i, with which the vector and the axial-vector currents are coupled in the QCD Lagrangian. u^μ is related to the covariant derivative of the pseudoscalar (Goldstone) fields. All these matrix valued fields transform only under the unbroken subgroup of QCD. Taking the trace denoted by $\langle\cdots\rangle$, the above terms are actually invariant under the full chiral symmetry group. Restricting to terms needed in our calculation, (2.12) gives immediately the vector and the axial-vector currents in the effective theory $[u\overleftrightarrow{\partial}_\mu v \equiv (\partial_\mu u)v - u\partial_\mu v]$,

$$V_\mu^i = F_\rho m_\rho \rho_\mu^i , \qquad A_\sigma^i = -g_1\epsilon_{\mu\nu\lambda\sigma}\omega^\mu\overleftrightarrow{\partial^\nu}\rho^{\lambda i},$$

and also the $\rho\omega\pi$ interaction,

$$\mathcal{L}_{\rho\omega\pi} = \frac{g_1}{F_\pi}\epsilon_{\mu\nu\lambda\sigma}\omega^\mu\overleftrightarrow{\partial^\nu}\rho^{\lambda i}\partial^\sigma\pi^i,$$

where the symbols denote the corresponding fields. Then we have for the matrix elements

$$\langle 0|V_\mu^i(0)|\rho^b(q,\sigma_2)\rangle = \delta^{ib}F_\rho m_\rho\epsilon_\mu(q,\sigma_2),$$
$$\langle\rho^b(q,\sigma_2)|A_\alpha^m(0)|\omega(q+k,\sigma_1)\rangle$$
$$= i\delta^{mb}g_1\epsilon_{\mu\nu\lambda\alpha}(2q+k)^\nu\epsilon^{\lambda*}(q,\sigma_2)\epsilon^\mu(q+k,\sigma_1),$$

where $\epsilon^\mu(q,\sigma)$ is the polarization state of the vector meson with momentum q and spin projection $\sigma = (1,0,-1)$. The remaining matrix element is a S-matrix element, which to first order in perturbation expansion is

$$\langle\omega(q+k,\sigma_1)|\pi^n(k'),\rho^c(q',\sigma_3)\rangle$$
$$= (2\pi)^4\delta(q+k-q'-k')\delta^{nc}i\frac{g_1}{F_\pi}\epsilon_{\mu\nu\lambda\sigma}(2q+2k-k')^\nu$$
$$\times k'^\sigma\epsilon^\lambda(q',\sigma_3)\epsilon^{\mu*}(q+k,\sigma_1).$$

Inserting these matrix elements in (2.11), we can carry out the sums over the polarization states of the vector mesons. After a little simplification, we get

$$\frac{1}{F_\pi}k^\alpha R_{\alpha\mu\nu}^{ij}$$
$$= -4i\delta^{ij}\left(\frac{g_1}{F_\pi}\right)^2\frac{\Lambda_{\mu\nu}}{(q^2-m_\rho^2)^2} \qquad (2.13)$$
$$\times\left\{\frac{1}{(q+k)^2-m_\omega^2} + \frac{1}{(q-k)^2-m_\omega^2}\right\},$$

where the Lorentz tensor $\Lambda_{\mu\nu}$ is given by

$$\Lambda_{\mu\nu}(q,k) = \epsilon_{\alpha\beta\gamma\mu}\epsilon_{\beta'\gamma'\nu}^\alpha q^\beta q^{\beta'}k^\gamma k^{\gamma'}$$
$$= (q.k)^2 g_{\mu\nu} + q^2 k_\mu k_\nu - (q.k)(k_\mu q_\nu + k_\nu q_\mu),$$

in the chiral limit ($k^2 = 0$). We can now write the complete result of our calculation as

$$i\int d^4 x e^{iq\cdot x}\text{Tr}e^{-\beta H}TV_\mu^i(x)V_\nu^j(0)/\text{Tr}e^{-\beta H}$$
$$= \left(1-\frac{4\mathcal{J}}{F_\pi^2}\right)i\int d^4 x e^{iq\cdot x}\langle 0|TV_\mu^i(x)V_\nu^j(0)|0\rangle$$

$$+ \frac{4\mathcal{J}}{F_\pi^2}\int d^4 x e^{iq\cdot x}\langle 0|TA_\mu^i(x)A_\nu^j(0)|0\rangle$$
$$+ 8\delta^{ij}\left(\frac{g_1}{F_\pi}\right)^2\frac{(q^2-m_\omega^2)}{(q^2-m_\rho^2+i\epsilon)^2} \qquad (2.14)$$
$$\times\int\frac{d^3 k\,n(k)}{(2\pi)^3 2|\boldsymbol{k}|}\frac{\Lambda_{\mu\nu}(q,k)}{(q^2-m_\omega^2+i\epsilon)^2 - 4(q\cdot k)^2},$$

where we reinstate $i\epsilon$ to define the poles and

$$\mathcal{J} = \int\frac{d^3 k\,n(k)}{(2\pi)^3 2|\boldsymbol{k}|} = \frac{T^2}{24}.$$

To proceed with the evaluation, we have to take note of the kinematics of the thermal two-point functions. Any such function can be decomposed into two invariant amplitudes. We now set $\boldsymbol{q} = 0$ with $q_0 \equiv E$, when the two amplitudes as functions of E become related to each other. Thus it suffices for us to consider, for example, the trace of the two-point function in its Lorentz indices. The free ρ pole term in vacuum then reads

$$i\int d^4 x e^{iqx}\langle 0|TV_\mu^i(x)V^{\mu j}(0)|0\rangle \xrightarrow{\rho} \frac{3\delta^{ij}(F_\rho m_\rho)^2}{E^2-m_\rho^2}. \quad (2.15)$$

Next we find this trace for the third term on the right in (2.14) in the vicinity of the ρ pole. If we ignore the mass difference between ρ and ω mesons, it can be immediately evaluated to reproduce the g_1^2 term in (1.1). In the realistic case with $\Delta m = m_\omega - m_\rho = 12\,\text{MeV}^1$, it is useful to note that the k-integral is essentially cut off at $|\boldsymbol{k}| = T$, because of the pion distribution function present in it. Thus if we are interested in the behavior of F_ρ^T for temperatures larger than Δm, as is usually the case, we can again ignore Δm to leading order to get (1.1). But for temperatures small compared to Δm, the k-integral is $O(T^4)$ and the g_1^2 term will not appear in (1.1).

3 Current algebra derivation of (1.2)

The properties of the nucleon can be studied by constructing a nucleon current $\eta_A^a(x)$ out of the quark fields, having the quantum numbers of the nucleon [4]. Here $a, b, \cdots$ and $A, B, \cdots$ denote respectively the isospin and the Dirac spinor indices. To study its properties at low temperature, we again expand, as in the case of vector currents, the thermal two-point function of nucleon currents,

$$\text{Tr}e^{-\beta H}T\eta_A^a(x)\bar{\eta}_B^b(0)/\text{Tr}e^{-\beta H}, \qquad (3.1)$$

in terms of the vacuum and the one pion state, and apply the method of current algebra to estimate the resulting pion matrix element of these currents,

$$\sum_m\langle\pi^m(k)|T\eta_A^a(x)\bar{\eta}_B^b(0)|\pi^m(k)\rangle, \qquad (3.2)$$

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[1] Working in the chiral limit, we should take Δm in this limit, which is even smaller than its physical value written above [9].

for small pion momenta in the chiral limit. As expected the calculation is similar to the earlier one with vector currents, except for the fermionic nature of the nucleon current.

The required Ward identity follows from the amplitude

$$
\begin{aligned}
Z^{mnab}_{\alpha,AB}&(p,k,p',k')\\
&= \int \mathrm{d}^4z\,\mathrm{d}^4x\,\mathrm{d}^4x'\,\mathrm{e}^{\mathrm{i}k\cdot z}\mathrm{e}^{\mathrm{i}p\cdot x}\mathrm{e}^{-\mathrm{i}p'\cdot x'}\\
&\quad\times\langle 0|TA^m_\alpha(z)\eta^a_A(x)\bar\eta^b_B(0)|\pi^n(k')\rangle.
\end{aligned}
\tag{3.3}
$$

Again using the axial-vector current conservation and the equal time commutation relation,

$$
\begin{aligned}
[A^m_0(z),\eta^a_A(x)]_{z_0=x_0}\\
= -\left(\frac{\tau^m}{2}\right)^{ac}(\gamma_5)_{AC}\eta^c_C(x)\delta^3(z-x)\,,
\end{aligned}
$$

we follow the steps as before to get the Ward identity,

$$
\begin{aligned}
k^\alpha Z^{mnab}_{\alpha,AB} &= -\mathrm{i}(2\pi)^4\delta^4(p+k-p'-k')\\
\times\Bigg[&\left(\frac{\tau^m}{2}\right)^{ac}(\gamma_5)_{AC}\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}(p+k)\cdot x}\\
&\times\langle 0|T\eta^c_C(x)\bar\eta^b_B(0)|\pi^n(k')\rangle\\
&+\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\\
&\times\langle 0|T\eta^a_A(x)\bar\eta^c_C(0)|\pi^n(k')\rangle\left(\frac{\tau^m}{2}\right)^{cb}(\gamma_5)_{CB}\Bigg].
\end{aligned}
\tag{3.4}
$$

The dominant contribution to $Z^{mnab}_{\alpha,AB}$ at small k,k' arises from the pion and the nucleon poles. Denote the latter contribution as

$$
Z^{mnab}_{\alpha,AB}\xrightarrow{N}(2\pi)^4\delta^4(p+k-p'-k')S^{mnab}_{\alpha,AB}(p,k,p')\,.
$$

The forward scattering amplitude (3.2) can now be obtained as

$$
\begin{aligned}
\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\sum_m&\langle\pi^m(k)|T\eta^a_A(x)\bar\eta^b_B(0)|\pi^m(k)\rangle\\
&= -\frac{3}{2F_\pi^2}\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\langle 0|T\eta^a_A(x)\bar\eta^b_B|0\rangle\\
&\quad -\frac{3}{2F_\pi^2}\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\langle 0|T\tilde\eta^a_A(x)\bar{\tilde\eta}^b_B|0\rangle\\
&\quad +\frac{1}{F_\pi}k^\alpha S^{ab}_{\alpha,AB}(p,k),
\end{aligned}
\tag{3.5}
$$

where $\tilde\eta=\gamma_5\eta$.

It remains to evaluate the contribution of the nucleon intermediate state to $Z^{mnab}_{\alpha,AB}$. Of course, it must be inserted twice more to couple it to the nucleon currents. We get

$$
\begin{aligned}
Z^{mnab}_{\alpha,AB}&\xrightarrow{N}\\
&\sum_{\sigma_1,\sigma_2,\sigma_3,c,d,e}\langle 0|\eta^a_A(0)|N^d(p,\sigma_2)\rangle\\
&\times\langle N^d(p,\sigma_2)|A^m_\alpha(0)|N^c(p+k,\sigma_1)\rangle\\
&\times\langle N^c(p+k,\sigma_1)|N^e(p',\sigma_3),\pi^n(k')\rangle\\
&\times\langle N^e(p',\sigma_3)|\bar\eta^b_B(0)|0\rangle\\
&\times\frac{\mathrm{i}^3}{(p^2-m_N^2)(p'^2-m_N^2)\{(p+k)^2-m_N^2\}}\\
&+\text{crossed term.}
\end{aligned}
\tag{3.6}
$$

The coupling of η to nucleon is given by the matrix element

$$
\langle 0|\eta^a_A(0)|N^d(p,\sigma)\rangle=\delta^{ad}\lambda u_A(p,\sigma)\,,
$$

where $u(p,\sigma)$ is the Dirac spinor of the nucleon. The piece in the effective Lagrangian $\mathcal{L}_{\mathrm{int}}=\frac{1}{2}g_A\bar\psi\slashed{d}\gamma_5\psi$ gives the nucleon contribution to the axial-vector current and the πNN coupling. We then have

$$
\begin{aligned}
\langle N^d&(p,\sigma_2)|A^m_\alpha(0)|N^c(p+k,\sigma_1)\rangle\\
&= \frac{1}{2}g_A\bar u(p,\sigma_2)\gamma_\alpha\gamma_5 u(p+k,\sigma_1)\chi^{d\dagger}\tau^m\chi^c,\\
\langle N^c&(p+k,\sigma_1)|N^e(p',\sigma_3),\pi^n(k')\rangle\\
&= (2\pi)^4\delta^4(p+k-p'-k')\left(-\frac{g_A}{2F_\pi}\right)\bar u(p+k,\sigma_1)\\
&\quad\times\slashed{k}'\gamma_5 u(p',\sigma_3)\chi^{c\dagger}\tau^n\chi^e,
\end{aligned}
$$

where the χ denote isospin states of the nucleon. Inserting these matrix elements in (3.6) and carrying out the spin and isospin summations, we get

$$
\begin{aligned}
S^{mnab}_{\alpha,AB}&\\
&= -\frac{\lambda^2 g_A^2}{4F_\pi}[(\slashed{p}+m)\gamma_\alpha(\slashed{p}+\slashed{k}-m)\slashed{k}'(\slashed{p}'+m)]_{AB}\\
&\quad\times\frac{\mathrm{i}^3}{(p^2-m_N^2)(p'^2-m_N^2)\{(p+k)^2-m_N^2\}}\chi^{a\dagger}\tau^m\tau^n\chi^b\\
&\quad +\text{ crossed term.}
\end{aligned}
\tag{3.7}
$$

We evaluate it in the forward direction for $\boldsymbol{p}=0$ and set $p_0\equiv E$ to get

$$
\begin{aligned}
\frac{1}{F_\pi}k^\alpha S^{ab}_\alpha\\
&= -6\delta^{ab}\left(\frac{\lambda g_A}{2F_\pi}\right)^2 p\cdot k(\slashed{p}+m)\slashed{k}(\slashed{p}+m)\frac{\mathrm{i}^3}{(p^2-m_N^2)^2}\\
&\quad\times\left(\frac{1}{p^2-m_N^2+2p\cdot k}+\frac{1}{p^2-m_N^2-2p\cdot k}\right)
\end{aligned}
\tag{3.8}
$$

$$
= -6\mathrm{i}\delta^{ab}\left(\frac{\lambda^2 g_A^2}{2F_\pi}\right)^2\frac{1}{E-m}\frac{1}{2}(1+\gamma_0),
\tag{3.9}
$$

in the vicinity of the pole. Inserting the results (3.5) and (3.9) into an equation similar to (2.1), the corrections change the free pole term in vacuum,

$$
-\frac{\lambda^2}{E-m+\mathrm{i}\epsilon}\frac{1}{2}(1+\gamma_0),
$$

to one with λ replaced by λ^T given by (1.2).

4 Discussion

In this work we use the method of current algebras to derive the temperature dependence of certain couplings, that have already been worked out with chiral perturbation theory [1,5]. Basic to the former approach is the extraction of terms of the appropriate amplitudes containing the axial-vector current operator, that are singular as the momentum carried by this operator goes to zero. Generally speaking, the use of PCAC can incorporate in the calculation only a part of such singular pieces, namely those due to the pion pole. But there can be other singular pieces in the amplitude from heavy particle poles also, that must be calculated separately. The calculation of the authors of [3] is incomplete in that it does not take the latter contribution into account, explaining the discrepancy between their results based apparently on current algebra and PCAC, and that of the effective theory.

In the case of the two-point function of vector currents, the contribution from the ω meson pole can be as dominant as the pion pole in the neighborhood of the ρ meson pole. To be specific, it brings in a small scale Δm in the expression. For temperatures small compared to Δm, the ω meson contribution is $O(T^4)$, changing the mass and residue only to this order. As T increases past a narrow transition region around $T = \Delta m$, the behavior changes over to $O(T^2)$, when F_ρ^T is given by (1.1). A similar result holds also for the axial-vector meson a_1 [1]. For the two-point function of nucleon currents, it is the nucleon pole that must be included. Here the situation is simpler, as there is no mass difference in the problem. Accordingly λ^T is given by (1.2) for all temperatures.

It is interesting to compare here this old method of current algebras to its modern development, namely that of effective field theory. The Ward identities used in the former method relate amplitudes with different numbers of pions, while in the effective theory one calculates the amplitudes directly in the perturbation expansion. When one is in need of vertices involving heavy particles in a current algebra calculation, one has, however, to turn to the effective chiral Lagrangian. The insertion of intermediate states is rather clumsy when compared with the use of propagators in the effective field theory. Also the two terms in (2.13) and (3.8) can be obtained neatly from the mass-shell delta function in the temperature dependent part of the pion propagator [1,5]. Thus even at the tree level, at which current algebra works, one cannot but appreciate the technical and conceptual superiority of the method of effective field theory.

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